

Example of finding extrema

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$$f(x,y) = 3x^2 - x^3 + 2xy + y^2$$

Find the critical points, local minima, local maxima, absolute minimum, absolute maximum, saddle points of f .

* Find critical points:

$$\begin{cases} f_x = 6x - 3x^2 + 2y = 0 \\ f_y = 2x + 2y = 0 \end{cases} \rightarrow y = -x \quad \begin{matrix} \curvearrowright \\ 4x - 3x^2 = 0 \end{matrix}$$

we get $\begin{cases} x = 0 \\ y = 0 \end{cases}$ and $\begin{cases} x = \frac{4}{3} \\ y = \frac{4}{3} \end{cases}$

* Second derivative test:

$$\left. \begin{array}{l} f_{xx} = 6 - 6x \\ f_{yy} = 2 \\ f_{xy} = 2 \end{array} \right\} D = f_{xx}f_{yy} - f_{xy}^2 = 8 - 12x$$

• At $(0,0)$: $D = 8 > 0$
 $f_{xx} = 6 > 0$

f attains local minimum at $(0,0)$.

• At $\left(\frac{4}{3}, -\frac{4}{3}\right)$: $D = -8 < 0$

$\left(\frac{4}{3}, -\frac{4}{3}\right)$ is a saddle point.

* Finding absolute min, absolute max:

$$f(0,0) = 0$$

0 is neither the abs. min. nor abs. max. of f because

$$f(1,0) < 0$$

$$f(-4,0) > 0$$

f has no abs. min. nor abs. max.